Non-Acceleration of Sgr A*: Implications for Galactic Structure

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ABSTRACT

We show that observations by Backer and collaborators over the past two decades constrain the time derivative of the proper motion of Sgr A* to be $< 0.14\,\mathrm{mas\,yr^{-2}}$. Using this result and a preliminary measurement by Eckart & Genzel of $\sigma \sim 500\,\mathrm{km\,s^{-1}}$ for the velocity dispersion of the star cluster within $0.\!\!''2$ of Sgr A*, we derive the following implications. First, if the nuclear star cluster is dominated by a massive black hole, then either Sgr A* is that black hole or it orbits the black hole with a radius $\lesssim 3\,\mathrm{AU}$. Second, even if the star cluster does not contain a massive black hole, Sgr A* is constrained to move slower than $20\,\mathrm{km\,s^{-1}}$ ($1\,\sigma$) relative to the center of mass of the cluster. The Galactocentric distance is therefore $R_0 = 7.5 \pm 0.7\,\mathrm{kpc}$, independent of the nature of Sgr A*. These error bars could be substantially reduced by future observations. If they are, it will also be possible to probe the motion of the nuclear star cluster relative to the center of mass of the Galaxy at the $\sim 4\,\mathrm{km\,s^{-1}}$ level.

Subject Headings: astrometry – Galaxy: center, fundamental parameters

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1. Introduction

Backer & Sramek (1987) and Backer (1996) have measured the proper motion of the compact non-thermal radio source Sgr A* and find values that are in reasonable agreement with those expected from the reflex motion of the Sun, assuming an LSR rotation speed $v_{\rm LSR} = 220\,{\rm km\,s^{-1}}$ and a galactocentric distance $R_0 = 7.5\,{\rm kpc}$. The $1\,\sigma$ error bars ($\sim 0.15\,{\rm mas\ yr^{-1}}$) correspond to $\sim 5\,{\rm km\,s^{-1}}$ at a distance of 7.5 kpc.

This agreement has been used as the basis for two related arguments. Backer (1996) reasons that since the source is moving at most very slowly relative to the proper motion of the Galactic center as predicted from "known" Galactic parameters, it must be a black hole of at least $100\,M_\odot$. On the other hand, Reid (1993) reasons that since the source is very likely to be at rest with respect to the center of mass of the Galaxy, one can use its proper motion to measure $v_{\rm LSR}/R_0$, and so (to the extent that $v_{\rm LSR}$ is considered known) constrain R_0. Each argument is important and interesting, but clearly both cannot be used together.

Here we show that upper limits on the *time derivative* of the proper motion of Sgr A* imply that it is very nearly at rest with respect to nuclear star cluster at the Galactic center. One may therefore use this proper motion to draw conclusions about Galactic structure independent of any assumptions about the nature of the source.

2. Observational Data

Comparing the results of Backer & Sramek (1987),

$$(\mu_l, \mu_b) = (-5.95 \pm 0.70, +0.43 \pm 0.50)\,{\rm mas\,yr^{-1}}, \qquad (1987), \tag{2.1}$$

and Backer (1996),

$$(\mu_l, \mu_b) = (-6.55 \pm 0.17, -0.48 \pm 0.12)\,{\rm mas\,yr^{-1}}, \qquad (1996), \tag{2.2}$$

it is clear that the proper motion of Sgr A* did not change much over a decade. It is difficult to give a precise upper limit to the time derivative of the proper motion because the underlying data have not been published. For purposes of this paper, we estimate the upper limit by combining equations (2.1) and (2.2) with the data points shown in Figure 1 of Backer & Sramek (1987) and find

$$\left[\left(\frac{d^2 l}{dt^2}\right)^2 + \left(\frac{d^2 b}{dt^2}\right)^2\right]^{1/2} < 0.14\,{\rm mas\,yr^{-2}}, \tag{2.3}$$

at the $1\,\sigma$ level, corresponding to a limit on the physical transverse acceleration $a_\perp$

of

$$a_\perp < a_{\rm max} = 5\,{\rm km\,s^{-1}\,yr^{-1}} \sim 0.025\,{\rm a}_\oplus, \tag{2.4}$$

where we have for simplicity of exposition adopted $R_0 = 7.5\,{\rm kpc}$, and where $a_\oplus$ is the acceleration of the Earth. We believe that this estimate is conservative, but in any event we indicate below how the results depend on $a_{\rm max}$.

From the work of Menten et al. (1997) the position on infrared images corresponding to the radio position of Sgr A* is now known to an accuracy of $0\overset{\prime\prime}{.}03$. Eckart & Genzel (1997) have measured the proper motions of stars in the infrared within $2''$ ($\sim 0.1\,{\rm pc}$) of this position. In general, they find that the velocity dispersion rises toward the center in a way that is consistent with a central black hole with mass $M_* = 2.45 \times 10^6\,M_\odot$. In particular, for the measurement at the innermost point at $r_* = 2000\,{\rm AU}$, they find a velocity dispersion $\sigma \sim 500\,{\rm km\,s^{-1}}$, i.e., still consistent with a central black hole of mass M_* at the center of the subregion of radius r_*,

$$M_* = 2.4 \times 10^6\,M_\odot, \qquad r_* = 2000\,{\rm AU}. \tag{2.5}$$

Eckart & Genzel (1997) regard the measurement at this last point as still preliminary. For purposes of this paper, we assume that a mass M_* is contained within a radius r_*, so that the magnitude of the acceleration at the boundary of this region is

$$a(r_*) = \frac{M_*/M_\odot}{(r_*/{\rm AU})^2} a_\oplus \sim 0.6\,a_\oplus. \tag{2.6}$$

As more data are acquired, it will be possible to refine the estimates of Eckart & Genzel (1997). The results presented here can then be rescaled.

Equations (2.4) and (2.6) reveal the basic result that we will exploit in this paper: the ratio, ϵ, of the upper limit for the transverse acceleration of Sgr A* to the characteristic acceleration of the system in which it is embedded is very small,

$$\epsilon \equiv \frac{a_{\rm max}}{a(r_*)} \sim 0.04\ . \tag{2.7}$$

To understand the implications of this result, we consider two limiting cases: first where the mass M_* is dominated by a single point mass (a black hole), and second where M_* is distributed uniformly throughout the region inside r_*. We demonstrate that in either case, Sgr A* is moving at most very slowly with respect to the center of mass of the star cluster in which it is embedded.

3. Kepler Potential

Suppose that the region within r_* is dominated by a massive black hole. Then there are two possibilities: either Sgr A* *is* that black hole or it is orbiting in the potential of the black hole. If the first is true, our case is already proved, so we restrict consideration to the second.

We designate the position of Sgr A* relative to the black hole by (r, θ, ϕ) where θ is the angle Sun–black hole–Sgr A*. Then,

$$\sin\theta = \frac{a_\perp}{a(r)} < \frac{a_{\max}}{a(r)} = \epsilon\left(\frac{r}{r_*}\right)^2. \tag{3.1}$$

The prior probability for such a fortuitous geometry at any given instant is extremely small, less than $(3/10)\epsilon^2 \sim 5\times 10^{-4}$ for a monotonically decreasing density profile. Even if Sgr A* happened to lie sufficiently close to the line of sight to the black hole at the beginning of the observations, it would move out of this zone within the $T \sim 10$ years of observations unless it were on a highly radial orbit, with its transverse speed $v_\perp$ constrained by $v_\perp < v_{\max} = \epsilon r^3/r_*^2 T \sim 40\,\mathrm{km\,s^{-1}}\,(\mathrm{r}/\mathrm{r}_*)^3$. This further reduces the prior probability by a factor $(v_{\max}/\sigma)^2/2$ to a net probability of $\lesssim 10^{-7}$. That is, this scenario is essentially ruled out.

Hence, if there is large black hole in the center of the nuclear star cluster, then Sgr A* must be it. The one potential loophole is that Sgr A* might be physically associated with the black hole and orbit it with a period much shorter than the frequency of observations, $\sim (450\,\mathrm{day})^{-1}$ (Backer & Sramek 1987). The physical association would evade the above probability argument, and the short period would imply that Sgr A* would orbit many times between observations and therefore would not show any secular acceleration. However, for an orbital radius $r \lesssim 150\,\mathrm{AU}$ (corresponding to a period $< 450\,\mathrm{days}$), the typical displacement between observations would be $\sim r/R_0 \sim 20\,\mathrm{mas}\,(r/150\,\mathrm{AU})$. The actual displacements from uniform motion are $\lesssim 5\,\mathrm{mas}$ (see Fig. 1 from Backer & Sramek 1987), implying that Sgr A* has an orbital radius $r \lesssim 40\,\mathrm{AU}$, and therefore a speed $v \gtrsim 7000\,\mathrm{km\,s^{-1}}$. Such large velocities are all but excluded by the observations of Rogers et al. (1994) who put an upper limit of 3.3 AU on the size of Sgr A* using observations taken on 1994 April 2 and 1994 April 4. The authors note that the observations were phased on NRAO 530 because the signal from Sgr A* was too weak. If Sgr A* had a transverse motion greater than $7000\,\mathrm{km\,s^{-1}}$, it would have moved more than 8 AU between the two sets of observations, which would have undermined the phasing and prevented Rogers et al. (1994) from setting an upper limit of 3.3 AU on the size (see Fig. 1 of Rogers et al. 1994). Only if $r < 3\,\mathrm{AU}$ (or if the orbital inclination and phase were particularly unfavorable) could this conclusion be avoided. However, even if Sgr A* were in such an orbit (as opposed to

being the black hole) its observed proper motion would still be equal to the proper motion of the black hole, since the size of the orbit would be a small fraction of the $\gtrsim 700\,\mathrm{AU}$ that Sgr A* has been observed to move relative to the Sun over the lifetime of the observations.

It is possible to test directly the hypothesis that Sgr A* is in a small orbit, $r \lesssim 3\,\mathrm{AU}$, by looking for time variability of the flux due to the Doppler effect. The fractional amplitude of the flux oscillations would be $f \sim (1+p)v\sin i/c$, where $p \sim 0.33$ is the slope of power law ($S_\nu \propto \nu^p$, Mezger 1996), i is the orbital inclination, and v is the orbital velocity. Thus $f \sim 0.12(r/3\,\mathrm{AU})^{-1/2}\sin \mathrm{i}$ with period $\sim 1.2\,\mathrm{days}\,(r/3\,\mathrm{AU})^{3/2}$.

4. Harmonic Oscillator Potential

Next, we suppose that the mass within r_* is not concentrated at a point, but rather is a distributed throughout the region, perhaps in the form of stars or possibly other objects. Most likely, the density profile would be monotonically decreasing, but for simplicity and to focus on an extreme case, we consider a uniform distribution. We note that there are many potential problems for a star cluster of this density because of the shortness of the relaxation time. However, the mass need not be in the form of stars, but could be in much lighter particles such as weakly interacting massive particles (WIMPs). Alternatively, the problems associated with dense star clusters might be avoided by some effect that has so far escaped recognition. Since our purpose is to develop completely general arguments, we do not make *any* assumption about the nature of the material within r_*, other than that it has total mass M_*.

A uniform distribution gives rise to a harmonic oscillator potential, so $a(r) = (r/r_*)a(r_*)$. Thus, the analog of equation (3.1) is

$$\rho = r\sin\theta = \frac{a_\perp}{a(r)}r < \frac{a_{\max}}{a(r_*)}r_* = \epsilon r_*, \tag{4.1}$$

where ρ is the projected separation of Sgr A* from the center of the cluster. The prior probability for this is $(3/2)\epsilon^2 \sim 2\times 10^{-3}$. As in the case of the Kepler potential, Sgr A* would have to be on a nearly radial orbit. Including both effects, the prior probability is $(3/4)\epsilon^4(r_*/\sigma T)^2 \sim 7\times 10^{-6}$. Again, the scenario is ruled out.

The one exception to this argument would be if Sgr A* were *gravitationally confined* to be near the center. Then it would not be at a random position in the cluster, and the previous probability argument would fail. In order to be

sufficiently confined to satisfy equation (4.1), $\rho < \epsilon r_*$, its characteristic speed would be constrained by

$$v \lesssim \epsilon\sigma \sim 20\,\mathrm{km\,s^{-1}}. \tag{4.2}$$

Thus even in this case, the proper motion of Sgr A* would be the same as that of the cluster center of mass to within $\sim 9\%$.

If the density profile fell monotonically (giving rise to a potential intermediate between Kepler and harmonic-oscillator) the arguments presented in this section would still hold but with greater force: the fraction of phase space satisfying the constraint (2.4) would be even smaller than 7×10^{-6}, and the maximum of velocity of an object gravitationally confined to a region that did satisfy the constraint would be even less than $20\,\mathrm{km\,s^{-1}}$.

We note in passing that by equipartition, the minimum mass of Sgr A* required for it to be gravitationally confined as described above is $M_{\mathrm{Sgr\ A*}} > \epsilon^{-2}M \sim 600\,M$, where M is the characteristic mass of the objects in the cluster.

5. Implications for Galactic Structure

Unfortunately, at the present time no hard and fast conclusions can be drawn from the lack of observed acceleration of Sgr A*. The arguments given above rest crucially on the Eckart & Genzel's (1997) observation of a high dispersion, $\sigma \sim 500\,\mathrm{km\,s^{-1}}$, within $0\overset{''}{.}2$ of Sgr A*. Since those authors regard their result as preliminary, any conclusions that are drawn from these observations must have the same caveat. Nevertheless, their preliminary result is quite plausible and could well be confirmed within a few years. We therefore begin by assuming that it will be confirmed and investigate the consequences.

Since equation (4.2) limits the motion of Sgr A* relative to the Galactic center to $< 20\,\mathrm{km\,s^{-1}} \sim 0.6\,\mathrm{mas\,yr^{-1}}$ at the $1\,\sigma$ level, one can apply the approach of Reid (1993) to constrain R_0 but without making any assumptions about the nature of Sgr A*. Of course, the price of relaxing these assumptions is the additional uncertainty in the motion of the Galactic center of $20\,\mathrm{km\,s^{-1}}$. To be specific, we adopt an estimate with $1\,\sigma$ error of $v_{\mathrm{LSR}} = 220 \pm 10\,\mathrm{km\,s^{-1}}$. We note in passing that the small size of this error bar rests critically on the assumption that the rotation curve of the Galaxy, like that of similar external galaxies, is flat. With this assumption, measurement of the redshifts of tangent points interior to the Sun lead to an estimate very close to $v_{\mathrm{LSR}} \sim 220\,\mathrm{km\,s^{-1}}$ (Brand & Blitz 1992). If this assumption is dropped, the error estimate increases by several fold. We assume that the Sun is moving at $12\,\mathrm{km\,s^{-1}}$ relative to the LSR, or $232\pm10\,\mathrm{km\,s^{-1}}$ relative

to the Galactic frame. We make use of Backer's (1996) measurement and $1\,\sigma$ error, $\mu_l = -6.55 \pm 0.17\,\mathrm{mas\,yr^{-1}}$ and find,

$$R_0 = 7.5 \pm 0.7\,\mathrm{kpc} \qquad \text{(provisional)}. \tag{5.1}$$

Even when the Eckart & Genzel (1997) measurement is confirmed, some of the most interesting information about Galactic structure will remain inaccessible due to the relatively weak upper limit (eq. (2.3)) to the time derivative of the proper motion. This limit can probably be significantly improved simply by fitting existing data to a second order polynomial. In any event, continued observations at a uniform rate and with uniform quality would yield a rapid improvement in the precision of this quantity, $\propto T^{-5/2}$, where T is the total duration of the observations. Hence, we also investigate what can be learned if the upper limit (2.3) can be significantly improved.

Of course, the uncertainty in equation (5.1) would be reduced. However, it would also be possible to probe an entirely different question: whether the nuclear star cluster at the Galactic center is at rest with respect to the center of mass of the Galaxy. At a distance of 7.5 kpc, Backer's (1996) proper-motion measurement in the b direction, $-0.48 \pm 0.12\,\mathrm{mas\,yr^{-2}}$ $(1\,\sigma)$, translates into $-17 \pm 4\,\mathrm{km\,s^{-1}}$. The Sun's motion relative to the LSR is $7\,\mathrm{km\,s^{-1}}$ and is extremely well measured, with an uncertainty of $\ll 1\,\mathrm{km\,s^{-1}}$ (Mihalas & Binney 1981). Hence there is a net motion of $-10 \pm 4\,\mathrm{km\,s^{-1}}$ that remains unexplained. At the present time, it is not possible to draw any conclusion about this residual for three reasons. First, the effect itself is detected at only the $2.5\,\sigma$ level and so could be just a statistical fluctuation. Second, the LSR may be moving relative to the Galactic frame because of a warp in the disk or some other effect. It is possible to directly test this hypothesis. Although the data available to date are inconclusive, significant improvements could be made in the future (see below). Third, the observed deviation is completely consistent with the constraint (4.2) on the motion of Sgr A* relative to the cluster. This limit is directly proportional to ϵ and so to $a_{\max}$ (see eq. (2.7)). If the upper limit on $a_{\max}$ could be reduced by a factor ~ 10, this would remove relative motion of Sgr A* as a potential cause of this effect. The same continued proper motion measurements would improve the statistical error on the proper motion in the b direction. Thus there would remain two potential causes, motion of the LSR and motion of the central star cluster (e.g. Miller & Smith 1992), both of which are interesting possibilities.

The motion of the LSR can be investigated by finding the mean motion of the Sun relative to stars in the Galactic halo. The velocity ellipsoids of several populations have been measured including 162 RR Lyraes (Layden et al. 1996;

Popowski & Gould 1997), 887 non-kinematically selected metal-poor field stars (Beers & Sommer-Larsen 1995), and 1352 high proper-motion stars (Casertano, Ratnatunga, & Bahcall 1990). However, only the authors of the RR Lyrae studies report on (or fit for) the mean z motion of their sample, $\langle W \rangle$. Popowski & Gould (1997) find $\langle W \rangle = -13 \pm 8\,\mathrm{km\,s^{-1}}$, which is consistent at the $1\,\sigma$ level with both the LSR value of $-7\,\mathrm{km\,s^{-1}}$ and the Sgr A* value of $-17\,\mathrm{km\,s^{-1}}$. It is straightforward to analyze the sample of Beers & Sommer-Larson (1995), since the data are publicly available and since the selection criteria are non-kinematic. We conduct a joint analysis of the Beers & Sommer-Larson (1995) stars and Layden et al. (1996) stars. To obtain a homogeneous sample, we restrict the former to stars within 3 kpc of the Sun and delete the variables (which are mostly RR Lyraes), and we restrict both samples to stars with [Fe/H] < -1.5. This leaves a total of 724 Beers & Sommer-Larson (1995) stars with radial velocities and 106 Layden et al. (1996) stars with both radial velocities and proper motions. Using the method of Popowski & Gould (1997) we find

$$\langle W \rangle = -6 \pm 5\,\mathrm{km\,s^{-1}}, \tag{5.2}$$

very close to the LSR value of $\sim -7\,\mathrm{km\,s^{-1}}$. It is truly unfortunate that Casertano et al. (1990) did not fit for $\langle W \rangle$ because at this point it would not be at all easy to reimplement the beautiful technique they devised to remove even unrecognized selection biases in their samples. If their samples were reanalyzed, however, we estimate that the error in the determination of $\langle W \rangle$ would be $\sim 5\,\mathrm{km\,s^{-1}}$. Thus, by combining the Casertano et al. (1990) sample with the results summarized in equation (5.2) based on the RR Lyraes and the non-kinematically selected stars, one could reduce the uncertainty to $\sim 3.5\,\mathrm{km\,s^{-1}}$. Hence, if the discrepancy persists between the z motion of the LSR and that of Sgr A*, it should be possible to decide which of them is actually moving relative to the Galactic center of mass.

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